

The Dynamic Accounting Multiplier from a SAM Perspective

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The purpose of this paper is to introduce the notion of the dynamic accounting multiplier, which could play a role in the empirical analysis of structural change analogous to the role played in static accounting multiplier analysis from a Social Accounting Matrix (SAM) perspective by the inverse of the flow coefficient matrix (Blancas 2006).¹

First, I shall summarize the static multiplier analysis that is used to point out the dynamic accounting multiplier approach. I shall describe the open dynamic system of the input-output model social accounting matrices in terms of a set of linear equations. Next, I shall present a general solution of that system, that is, the inverse of its structural matrix. Each element of this inverse represent the combined direct and indirect inputs required from the row institutional sector to create an additional output of 1 million by the column institutional sector.

Since the accounting multiplier analysis developed in this paper is focused on financial transactions (Blancas, 2002), the dynamic of the cash flow between accounts is given by the quantity spent by the account j in the account k at year t . This cash flow will turn into a profit or a loss at the end of every year. Such a dynamic version is different from the traditional input/output matrix analysis, where the dynamics is given in terms of the physical investment. The common methodology in mathematical economics to explain the dynamics of a specific

¹ This static accounting multiplier analysis is termed also as static Interinstitutional Linkage Analysis.

model is to start from the static version and then expose the dynamics of the model. This paper runs in the same track: in the first section we point out some characteristics of the static version of an accounting multiplier analysis derived from a SAM. Section 2 shows the dynamic version. The last section displays some conclusions.

1. Static Multipliers Analysis

Let S be a normalized social accounting matrix with two subgroups: real accounts and imaginary accounts. Then S can be rearranged as follows:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where S_{11} stands for the cash flow at a given year between real accounts, real intraflow, (households, intermediate demand...), S_{22} is the cash flow between the imaginary accounts, imaginary intraflow. S_{12} and S_{21} are investment and savings respectively. After selecting some accounts as exogenous, any, we will proceed to rearrange S , the respective rows are taken into the leak matrix, and the columns are left out of the endogenous block to account for injections and intraexogenous cash flows.

$$S = \begin{bmatrix} \text{[Intraendogenous Flows]} & \text{[Injections]} \\ \text{[Leaks]} & \text{[Intraexogenous Flows]} \end{bmatrix}$$

After this reclassification of exogenous and endogenous accounts, the submatrix [Intraendogenous Flows] is still a square matrix, since we removed out of it the same number of columns and rows, but still accounts belong to either subgroup.

$$\text{[Intraendogenous Flows]} = S = \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} \text{ where } S_{ij}^* \text{ is understood exactly as } S_{ij}, i, j \in \{1,2\}$$

but exclusively between endogenous accounts.

We may wonder, how would the intraendogenous cash flow modify, if money were injected into the system through [Injections]? Which clearly leads us to usual Leontief's inverse analysis. Using the fact that total income equals intraendogenous accounts plus injections², we obtain:

$$y = S^* y + X e = S^* y + x \quad \dots(1)$$

$$\Rightarrow y = (I - S^*)^{-1} X e$$

$$\text{or } y = (I - S^*)^{-1} x = Mx \text{ donde } X i = x$$

^te = (1 1 ... 1), dim (e) = # exogenous accounts. Then $(I - S^*)^{-1}$ is a typical

multipliers matrix. However, at first glance, it's still impossible to discern the

² In the input-output context we say that total production equals intermediate demand plus final demand.

injection's path through the accounts in S^* . We decompose S^* , based in a selection of endogenous accounts of more interest. In our case this subgroups will be the endogenous real accounts and the endogenous imaginary accounts.

This leads us to an additive decomposition of S^* as follows:

$$S^* = \begin{bmatrix} S_{11}^* & 0 \\ 0 & S_{22}^* \end{bmatrix} + \begin{bmatrix} 0 & S_{12}^* \\ S_{21}^* & 0 \end{bmatrix}$$

Let $B = \begin{bmatrix} S_{11}^* & 0 \\ 0 & S_{22}^* \end{bmatrix}$ and $C = \begin{bmatrix} 0 & S_{12}^* \\ S_{21}^* & 0 \end{bmatrix}$, then (1) looks like:

$$y = (B + C) y + x$$

where after recursively substituting, we obtain:

$$\begin{aligned} y &= (I - B)^{-1} C y + (I - B)^{-1} x \\ &= [I - (I - B)^{-1} C]^{-1} [I - B]^{-1} x \end{aligned}$$

Let $(I - B)^{-1} C = R$, then

$$[I - (I - B)^{-1} C]^{-1} = (I - R)^{-1}; \text{ Analogously, from the geometric series}$$

$$(1 - r)(1 + r + r^2 + \dots + r^j) = 1 - r^{j+1} \therefore \frac{1}{1 - r} = (1 - r)^{-1} = \frac{1 + r + \dots + r^j}{1 - r^{j+1}} = (1 - r^{j+1})^{-1} (1 + r + \dots + r^j)$$

Then

$$y = [I - R^{j+1}]^{-1} [I + R + \dots + R^j] [I - B]^{-1} x = M_3 M_2 M_1 x = M x^3$$

³ For $n=2$ we would obtain the classic decomposition, the value of n depends on the additional information from M_2 and M_3 as well as their economic meaning.

This is a multiplicative decomposition of Leontief's inverse, where we can observe the trajectory of the injection, as long as the inverse matrices exist [1].

$$\text{Since } (I - B)^{-1} = \begin{bmatrix} (I - S_{11}^*)^{-1} & 0 \\ 0 & (I - S_{22}^*)^{-1} \end{bmatrix}$$

We have clearly, obtained the partial intragroup multiplier effect of an exogenous injection to the system. We could define M_2 as the spillover effects matrix and M_3 as the Feedback one, but these are not unique! [2]. Round comments that in his experience, in practice M_3 adds no information. Suppose that R^3 is irrelevant then the sum process stops at $n=2$ and M_3 is almost the identity matrix, since $M_3 = I - R^3$.

So far we have only discerned the path the injection follows through the endogenous subgroups, (with the aid of the multiplicative decomposition).

$My = y + \Delta y$, and we are only interested in the net effect Δy we write $M_3 M_2 M_1$ as a telescopic sum to obtain

$$\begin{aligned} \Delta y &= M_3 M_2 M_1 y - y = [(M_3 M_2 M_1 - M_2 M_1) + (M_2 M_1 - M_1) + (M_1 - I) + I] y - y \\ &= [(M_3 - I) M_2 M_1 + (M_2 - I) M_1 + (M_1 - I)] y \\ &= (N_3 + N_2 + N_1) y \end{aligned}$$

Which is an additive decomposition of Leontief's inverse, where we can observe

the final destination of the injection. N_1 = net intragroup effects, N_2 := net spillover effects, N_3 : net feedback effects.

3. Dynamic Model

In the traditional input/output model the b_{tji} ($X_{t+1i} - X_{ti}$) terms represent the capital goods produced by sector j at time t required for the production of sector i in time period $t + 1$ (Leontief, 1986).

In our example the SAM consists of real and financial accounts. Since the dynamic of the cash flow between these accounts is the same, I will study them all equally, whatever the nature of the accounts may be. Let $s_{kj}(t)$ be the quantity spent by account j in account k at year t . This cash flow will turn into a gain or a loss at the end of every year. Then

$$(1 + \tau_k)s_{kj}(t) = s_{kj}(t) + \tau_k s_{kj}(t) = q_{kj}(t+1) \quad (2)$$

is a quantity available only at the end of year t , where τ_k is the interest rate offered by imaginary account k .

This rate could be negative whenever we registered a loss in a fixed year.

Since our system is in equilibrium and closed, just into the SAM that includes the foreign sector, $q_{kj}(t+1)$ must be spent or reinvested completely back into the system.

Obviously this flow must be registered in $S(t+1)$ (the SAM for year $t+1$) therefore

$$\beta_k \sum_{i=1}^m s_{ij}(t+1) = q_{kj}(t+1) \quad 0 < \beta_k \leq 1$$

$$k = 1, \dots, m \quad (m \text{ endogenous accounts})$$

where $\sum_k \beta_k = 1$. Notice that there exists at least one k such that $\beta_k > 0$ since our

system is closed. Hence from (2) we obtain:

$$\beta_k \sum_{j=1}^m s_{ij}(t+1) = (1 + \tau_k) s_{kj}(t), \quad k=1, \dots, m. \quad (3)$$

This equation (3) accounts for: a part of the total expenditure of account j at time $t+1$ equals the revenue of the investment of account j into account k at time t .

If we sum the last m identities over k we will obtain the consistent identity:

$$\sum_{k=1}^m (\beta_k \sum_{i=1}^m s_{ij}(t+1)) = \sum_{k=1}^m (1 + \tau_k) s_{kj}(t), \quad \text{but factoring out}$$

$$\sum_{k=1}^m (\beta_k \sum_{i=1}^m s_{ij}(t+1)) = (\sum_{i=1}^m s_{ij}(t+1)) (\sum_{k=1}^m \beta_k) = \sum_{i=1}^m s_{ij}(t+1), \quad \text{therefore}$$

$$\sum_{i=1}^m s_{ij}(t+1) = \sum_{k=1}^m (1 + \tau_k) s_{kj}(t).$$

This last equation simply states that whatever account j gained at the end of year t will be **accounted as expenditure of j in the SAM at the next year.**

The m equations in **(3)** describe whole dynamic of each account $j=1,\dots,m$. Hence we just obtained a system of m^2 equations that describe the dynamic of the Social Accounting Matrices through time.

Usually the vector $(\beta_1^j, \dots, \beta_m^j)$ with $\sum_{k=1}^m \beta_k^j = 1$, $j=1,\dots,m$ is called a portfolio. At

the beginning of investment it is called a benchmark. This one must be always chosen to be risk free. Since we could always start at any year, every SAM should be risk free. Therefore we could measure some structural instability or stability of an economy, measuring the risk of each of its m portfolios β^j .